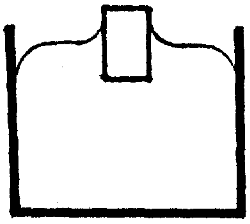

Fig. 3.

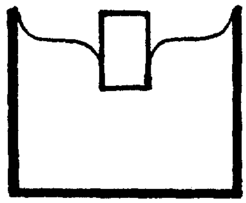

Fig. 4.

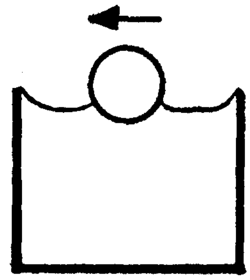

Fig. 5.

as shown in Fig. 4. Difficulties arise with this solution because most objects do not have the idealized shape shown in Fig. 4. The angle of contact between the water and the object's sides is primarily dependent upon the composition of the material, not the angle that the object's sides make with the horizontal. Thus, a well waxed ping-pong ball will produce a water surface like that shown in Fig. 5 rather than one like that shown in Fig. 4.

A different approach to the problem is to use a lack of surface tension to keep the object in the center. If soap is added to the water the object will wet better. Wetted objects drag water along with them when they move while objects which are not wetted move through the water with less obstruction. The friction that a wetted object experiences is sufficient to keep it from moving to the edge of a glass unless the object is already very near the edge.

Ignoring the whole question of surface tension there are other solutions. One is to stir the water. If the water is whirling around in the glass any floating objects will move toward the center. If it is not legitimate to stir the water the same thing can be accomplished by placing the glass on a turntable.

Another solution involves Bernoulli's principle. Simply pour water directly on the object while it is in the center of the glass. The stream of water will hold the object in the center. Ping-pong balls work especially well for this situation.

Still another, but probably not the final solution, is to hold a charged comb directly over the object. Here again ping-pong balls work well.


**Reference**

1. James T. Schreiber, "Barroom Physics Part II," Phys. Teach. 13, 418 (1975).


# Stepwise approximation to an orbit (revisited)


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*The Project Physics Course Handbook*[1] describes an activity entitled "Stepwise Approximation to an Orbit" in which the student plots the motion of a comet over a series of short time intervals, applying Newton's laws of motion and gravitation at each step. The resulting approximate orbit may, in principle, be used to verify Kepler's laws of planetary motion, illustrating the fact that Kepler's laws follow from Newton's laws.

In concept the activity is excellent, but the iterative procedure which is described leads to disappointing results. Students typically feel frustrated that increased care fails to improve their orbits. The procedure can be modified, however, to produce quite good results.

We begin with the approximation that over short intervals the gravitational force felt by a comet is constant in magnitude and direction. Over each interval, then, the comet undergoes simple projectile motion. Its displacement in the nth interval is

$$\Delta\vec{r}_n = \vec{v}_n\,\Delta t + \tfrac{1}{2}\,\vec{g}(\vec{r}_n)\,(\Delta t)^2 \qquad (1)$$

which we shall abbreviate $r_n = r_{(n)\mathrm{inert.}} + r_{(n)\mathrm{grav.}}$. The final velocity for the nth interval, which is also the initial velocity for the following interval, is

$$\vec{v}_{n+1} = \vec{v}_n + \vec{g}(r_n)\,\Delta t \qquad (2)$$

Those familiar with the derivation presented in *The Project Physics Course Handbook* will recognize the velocity given in Eq. (2) as the velocity said to be maintained *throughout* the nth interval. The displacement that results is

$$\Delta\vec{r}_n = \vec{v}_n\,\Delta t + \vec{g}(r_n)\,(\Delta t)^2 \qquad (3)$$

which is clearly in error. Moreover, the gravitational component of the displacement is inadvertently omitted in the first iteration. Figure 1 shows the results of the *Project Physics* version of the activity both with and without the initial gravitational term. In both cases the comet deviates from the theoretical orbit immediately and the period is at

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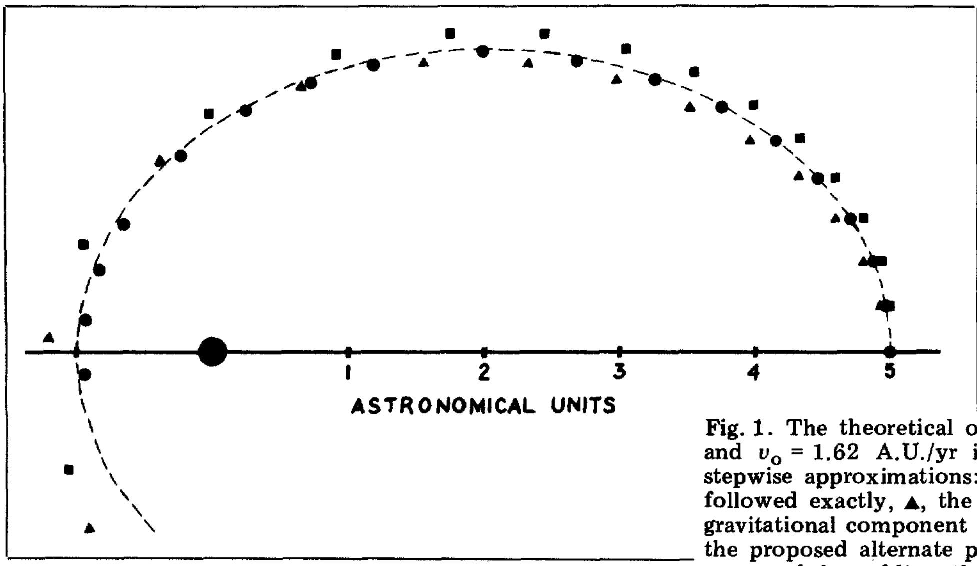


Fig. 1. The theoretical orbit of a comet with $r_o$ = 5 A.U. and $v_o$ = 1.62 A.U./yr is compared with three different stepwise approximations: ■, the *Project Physics* procedure followed exactly, ▲, the *Project Physics* procedure with a gravitational component added in the first iteration, and ●, the proposed alternate procedure. The respective periods, measured by adding the iteration intervals, are 5.54 yr, 4.96 yr, and 5.19 yr. The theoretical period is 5.20 yr. In the alternate procedure the first ten iterations use $\Delta t$ = 0.205 yr. The next four use $\Delta t/2$, and the last three use $\Delta t/4$. All three stepwise approximations were computed, rather than graphically constructed, to allow comparison of the intrinsic errors alone.

variance with Kepler's third law. The errors to converge to zero as $\Delta t$ approaches zero, but the convergence is too slow to produce satisfactory results for hand-generated orbits. It is noteworthy that the *Project Physics Course* finally relies on filmloops of computer-generated orbits to make its point.

Equations (1) and (2) can be translated directly into a graphical iterative procedure. To avoid repeated computations we first represent $\Delta r_{\text{grav.}}$ graphically as a function of $r$. From Newton's law of gravitation it follows that

$$\vec{g}(r) = -(GM/r^2)\hat{r}. \tag{4}$$

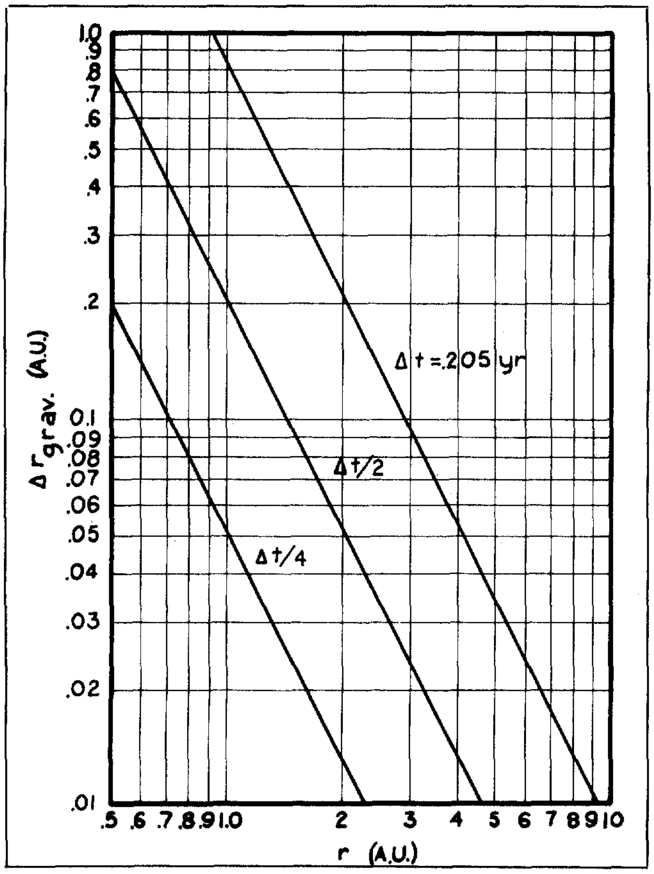


Fig. 2. The function, $\Delta r_{\text{grav.}} = k/r^2$, is plotted on log-log paper for the primary interval $\Delta t$ = 0.205 yr, and for supplementary intervals $\Delta t/2$ and $\Delta t/4$.

By substituting into Eq. (1) we find

$$\Delta\vec{r}_{\text{grav.}} = -(\tfrac{1}{2}GM(\Delta t)^2/r^2)\hat{r} = -(k/r^2)\hat{r}, \tag{5}$$

where $k$ is constant if $t$ is held fixed. The function $\Delta r_{\text{grav.}} = k/r^2$ can be plotted as a straight line on log-log paper as in Fig. 2. (For our purposes the most convenient units of length, mass, and time are the astronomical unit, the solar mass, and the year. In these units $G = 4\pi^2\,(\text{A.U.})^3 M_o^{-1}\text{yr}^{-2}$.)

The actual plotting of the orbit may now begin as in Fig. 3. Choosing the initial position, $A$, and velocity, $\vec{v}_o$, we construct $AB = \Delta\vec{r}_{(\text{o})\text{inert.}}$. Measuring the distance from the sun, $S$, to the approximate midpoint of $AB$ we find the corresponding value of $\Delta r_{(\text{o})\text{grav.}}$, and plot $BC = \Delta r_{(\text{o})\text{grav.}}$ along the line $BS$. $C$ becomes the initial position for the second iteration.

We may find $\Delta\vec{r}_{(1)\text{inert.}}$ by multiplying Eq. (2) through by $\Delta t$.

$$\Delta\vec{r}_{(1)\text{inert.}} = \vec{v}_o\Delta t + \vec{g}(r_o)\,(\Delta t)^2$$

$$= \Delta\vec{r}_{(\text{o})\text{inert.}} + 2\Delta\vec{r}_{(\text{o})\text{grav.}} \tag{6}$$

In Fig. 3a, $AX$ is the desired vector. If we make the approximation that $SA \| SB$, an approximation already involved in treating the vector $\vec{g}(r_n)$ as a constant, a vector equal to $AX$ originating at $C$ can be constructed as shown in Fig. 3a. Having constructed $\Delta\vec{r}_{(1)\text{inert.}}$, we can evaluate $\Delta\vec{r}_{(1)\text{grav.}}$ and continue the process as in Fig. 3b.

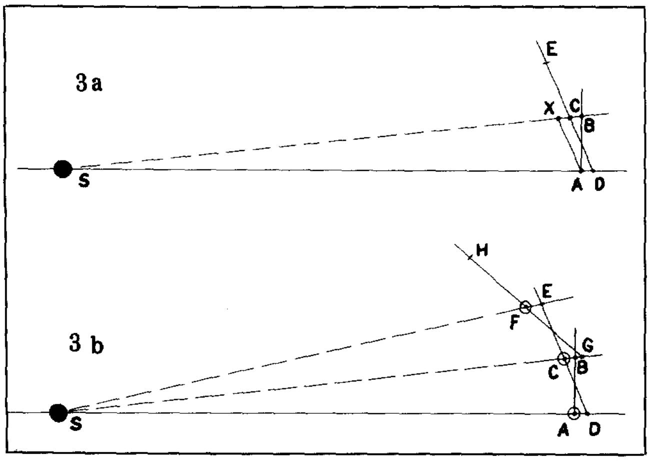


Fig. 3. The first two iterations using the alternate procedure are illustrated. $A$ is the initial position, $AB = \vec{v}_0 \Delta t$, and $BC = \Delta\vec{r}_{(o)grav.}$. $AX = \Delta\vec{r}_{(1)inert.}$. To construct $CE = AX$, plot $D$ on $SA$ such that $AD = BC$. Extend line $DC$ and plot $E$ such that $CE = DC$. Note that in practice $X$ need not be plotted. The second iteration continues by plotting $EF = CG = \Delta\vec{r}_{(1)grav.}$, extending $GF$, and plotting $FH = GF$. $A$, $C$, and $F$ lie on the approximate orbit.

For orbits of any appreciable eccentricity the displacement per time interval may become so large as the comet approaches the sun that the constant field assumption no longer holds. When this happens, $\Delta t$ can be divided in half by bisecting $\Delta\vec{r}_{inert.}$ and evaluating $\Delta\vec{r}_{grav.}$ from a second graph based on the interval $\Delta t/2$. The family of functions based on $\Delta t$, $\Delta t/2$, $\Delta t/4$, . . . , are represented on log-log paper by equally spaced parallel lines (see Fig. 2).

To evaluate the results, the theoretical orbital parameters must be known in terms of the initial conditions. If the initial velocity is chosen perpendicular to the initial position vector the eccentricity is given by

$$e = |(v_o^2 r_o/GM) - 1| \tag{7}$$

The value of $e$ determines which conic section will be produced. The initial position is the perihelion or aphelion point according to whether $v_o^2$ is greater than or less than $GM/r_o$. For elliptical orbits the semimajor axis, $a$, is given by $r_o/(1+e)$ or $r_o/(1-e)$ depending on whether $\vec{r}_o$ is the aphelion or perihelion point. The theoretical period may be determined from Kepler's third law: $T^2/a^3 = K$, where $K = 1/M$ in our unit system.

Superimposing the theoretical orbit on the stepwise approximation is a good exercise for the student. The intrinsic errors of the method we have presented are small enough that a comparison with the theoretical orbit essentially indicates the degree of graphical precision. Careful students are thereby rewarded for their efforts.

**Reference**

1. F. James Rutherford, Gerald Holton, and Fletcher G. Watson, *The Project Physics Course Handbook* (Holt, Rinehart and Winston, New York, 1970), pp. 114-118. The experiment in *The Project Physics Course Handbook* was inspired by a similar, but more mathematically complex experiment described by Dr. Leo Lavatelli in Am. J. Phys. 33, 605 (1965).

# A fun experiment with Newton's second law

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While preparing for a laboratory for two year technology degree students, we came up with an experiment involving Newton's second law which was fun and straightforward but which has not previously been published in this journal. The time had arrived for an experiment to demonstrate this law. Thoughts of Atwood's machines, air tracks, and little carts were floating through our minds. But surely students believe Newton's second law. Hasn't everything else which we have told them been true as proven by us in lecture demonstration or themselves in laboratory? Let's not prove Newton's law! Surely the physics class did last year; probably even Newton did. Instead why not use Newton's second law to measure the mass of some object?

Once again thoughts of gliders, weights, and pulleys started flying through our minds. No! Stop! Let's get away from this equipment designed mainly for a physics laboratory which the student will most likely never see again and take the experiment to his environment. Yes, it was finally clear: the students should measure the mass of something in their environment by pushing or pulling it with a known force. They could measure the resulting acceleration and calculate its mass with the help of Newton's second law. Probably the object should be something large so it would not accelerate too fast or could not easily be placed on a scale.

A car was the obvious answer. In clear, warm weather a car belonging to one of the instructors, a Datsun 240Z, was chosen. The location of the experiment was one of the parking lots on campus (Fig. 1). At other times the object of interest was a laboratory cart, or the instructor's chair. In these cases the experiment was done in the hallway outside of the physics lab (Fig. 2). Since all of these objects had wheels, the frictional forces should remain relatively constant and presumably could easily be taken into account.